Online Appendix

A Behavioral Definition of Unforeseen Contingencies

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In this appendix I extend the analysis of the paper in several directions. First, I discuss two additional restrictions on behavior that can further discipline the model without ruling out unforeseen events. Second, I extend the analysis to a fully-fledged dynamic setting. Finally, I study repeated choice over time and propose a relaxation of Dynamic Consistency that is compatible with unforeseen events. The proposed axiom implies that foresight expands over time and delivers a rule for updating beliefs.

A Additional Restrictions on the Model

In this section, I continue to maintain the simple three-period setting from the paper. Recall the first part of Boundedness which required that $d^b A h \succeq h \succeq d^w A h$ for every minimal event $A \in \mathcal{F}$ and every act $h \in \mathcal{H}$. The implied bounds on the evaluation of h are far from demanding. To see this, note that the consumption streams $d^b, d^w \in X^\infty$ are not specific to the act h; in fact, they are the best and worst streams among *all feasible* streams $d \in X^2$. Tighter bounds can be imposed if the individual has more specific understanding of the objective environment. Indeed, suppose as in Section 2 that the individual knows the set of possible outcomes within each coarse atom and each period. Write $x \succeq^* y$ if $(x,x) \succeq (y,y)$ and $h \succeq^* g$ if $h_t(\omega) \succeq^* g_t(\omega)$ for every t and ω. One can interpret $x \succeq^* y$ as $x \in X$ being a better outcome than $y \in X$, and $h \succeq^* g$ as the respective pointwise order on $\mathcal{H}$. Given an act $h \in \mathcal{H}$ and a minimal event $A \in \mathcal{F}$, let $h^b_t(A) \in X$ and $h^w_t(A) \in X$ be the best and, respectively, the worst

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outcomes in $h_t(A) \subset X$ according to the order $\succeq^*$. Letting $d^b_{h,A} := (h^b_1(A), h^b_2(A)) \in X^2$ and $d^w_{h,A} := (h^w_1(A), h^w_2(A)) \in X^2$, one can then require that

$$d^b_{h,A} A h \succeq h \succeq d^w_{h,A} A h.$$

Strengthening Boundedness in this manner is equivalent to having a proper small world representation with the mapping Φ satisfying $\Phi^l \leq \Phi \leq \Phi^u$.

One can strengthen Boundedness further by imposing **Pointwise Monotonicity**, which requires that $h \succeq g$ whenever $h \succeq^* g$. In terms of the representation, the axiom implies that Φ is increasing in the pointwise order $\succeq^*$ on $\mathcal{H}$.[1] The first thing to note about this axiom is that it does not rule out unforeseen events in the sense of Definition 1: the mapping Φ^l in Example 1 is increasing. Roughly, this is because the axiom, being a pointwise condition, has less to do with the autocorrelations that are central to Definition 1. Compare this with the discussion of Ordinal Dominance in Section 12.

Pointwise Monotonicity may provide useful discipline in applications in which the outcomes are monetary ($X \subset \mathbb{R}$) and $x \succeq^* y$ means simply that more money is better. To the extent that all individuals agree on the latter, one may then worry that an individual who does not foresee all contingencies can be exploited and driven out of the market. Pointwise Monotonicity makes such exploitation difficult by insuring that no single decision of the individual will result in a certain loss. Apart from specific examples however, such as Example 1, it is hard to pin down how well and in what ways the individual should understand the objective environment in order for Pointwise Monotonicity to hold. This is why I did not impose the axiom in the main text.

B A Multi-Period Model

As in Section 10, suppose time is discrete and varies over an infinite horizon: $t \in \{0, 1, 2, ...\} =: T$. The arrival of information is modeled by a filtration $\mathcal{A} = \{\mathcal{A}_t\}_t$, that is, a sequence of algebras such that $\mathcal{A}_0 = \{\varnothing, \Omega\}$ and $\mathcal{A}_t \subset \mathcal{A}_{t+1} \subset 2^\Omega$ for every t. As is standard in the analysis of repeated choice, assume that each algebra $\mathcal{A}_t$ is finite and let $\mathcal{A}_t(\omega)$ be the smallest event in $\mathcal{A}_t$ that contains $\omega \in \Omega$. An **objective act** h, or simply an **act**, is an X-valued, $\mathcal{A}$-adapted process, that is, a sequence $(h_0, h_1, ...)$ such that $h_t : \Omega \to X$ is $\mathcal{A}_t$-measurable for every t. An act h is **finite** if there is some $t \in T$ such that h_k is $\mathcal{A}_t$-measurable for every $k \in T$. Let $\mathcal{H}$ be the space of all finite acts h and let $\succeq$ be a preference relation on $\mathcal{H}$ representing ex ante behavior.

[1] More formally, Φ is increasing if $h \succeq^* g$ means that $[\Phi h]_t(\omega) \geq [\Phi g]_t(\omega)$ for every $t \in \{1, 2\}$ and $\omega \in \Omega$.

C A Behavioral Definition of Foreseen Events

This section provides an analogue of Definition 1. The main difference is that uncertainty may resolve gradually over time. This means that the new definition has two tasks. First, it must elicit whether an event A is foreseen. Second, if the event A is foreseen, the definition must elicit the time period in which the individual believes the event A is going to resolve. Consequently, instead of a single collection $\mathcal{F} \subset \cup_t \mathcal{A}_t$, the definition delivers a sequence $\{\mathcal{F}_t\}_t$ of such collections, with each $\mathcal{F}_t \subset \mathcal{A}_t$ collecting the events which the individual foresees and which he believes to resolve before or in period t. Some preliminary concepts are needed to state the definition. An act $h \in \mathcal{H}$ is **effectively certain** if $h(\omega) \sim h(\omega')$ for every $\omega, \omega' \in \Omega$. An effectively certain act $h \in \mathcal{H}$ is **subjectively certain** if $h \sim h(\omega)$ for some $\omega \in \Omega$. Given $t \in T, A \in \mathcal{A}_t$, and $d, d' \in X^\infty$, let $dA^t d'$ be the act $h \in \mathcal{H}$ such that $h_k = d'_k$ for every $k < t$ and $h_k = d_k A d'_k$ for every $k \geq t$.[2]

Definition C.1 *For every $t \in T$, let $\mathcal{F}_t$ be the collection of all events $A \in \mathcal{A}_t$ such that every effectively certain act of the form $dA^t d', d, d' \in X^\infty$, is subjectively certain.*

It follows directly from the definition that $\mathcal{F}_t \subset \mathcal{F}_{t+1}$ for every t. Also, each collection $\mathcal{F}_t$ is closed with respect to complements and includes the event Ω. If an analogue of Coarse Recursivity is imposed, one can show that each collection $\mathcal{F}_t$ is an algebra. Letting $\mathcal{F} := \{\mathcal{F}_t\}_t$, one can think of $\mathcal{F}$ as the individual's subjective event tree.[3]

D A Small-World Representation

The section extends the notion of a small-world representation to the multi-period setting of this appendix. The main difference is that one has to work with a filtration $\{\mathcal{G}_t\}_t$ instead of a single algebra. Accordingly, let $\mathcal{G} = \{\mathcal{G}_t\}_t$ be a filtration on Ω such that $\mathcal{G}_t \subset \mathcal{A}_t$ for every t. Let $u : X \to \mathbb{R}$ be a utility index over outcomes

[2] As in the paper, the space X^∞ is identified with the space of deterministic acts in $\mathcal{H}$. Given $t \in T$ and $d \in X^\infty$, $d_t \in X$ denotes the t^{th}-element of the sequence. Thinking of d as an act, d_t becomes a constant function from Ω into X. Given an act $h \in \mathcal{H}$ and a state $\omega \in \Omega$, $h(\omega)$ denotes the sequence $(h_0(\omega), h_1(\omega), ...)$.

[3] It is possible that an event $\mathcal{A} \in \mathcal{A}_t$ belongs to $\mathcal{F}_{t+k}$ for some $k > 0$ but not to $\mathcal{F}_t$. This is a situation in which the individual foresees the event A but believes that the event resolves later than it actually does. In Kochov [6], I give a restriction on behavior that insures that the individual does not misjudge the timing of any foreseen event. See the discussion of Path Stationarity on p.16 of that paper.

and $\beta \in (0,1)$ a discount factor.[4] Let $\mathcal{R}(u,\mathcal{G})$ be the space of all $u(X)$-valued, $\mathcal{G}$-adapted processes on Ω. A typical element of $\mathcal{R}(u,\mathcal{G})$ is a sequence $(\hat{h}_0, \hat{h}_1, ...)$ where each $\hat{h}_t : \Omega \to u(X)$ is a $\mathcal{G}_t$-measurable function. Given $h \in \mathcal{H}$, $u \circ h$ denotes the process $(u \circ h_0, u \circ h_1, ...) \in \mathcal{R}(u,\mathcal{A})$. The next definition, which parallels Definition 2, introduces the notion of a $\mathcal{G}$-approximation mapping Φ from the space of objective acts into the space of subjective acts. To state the definition, endow $\mathcal{H}$ and $\mathcal{R}(u,\mathcal{G})$ with the respective product topologies.

Definition D.1 *A continuous function $\Phi : \mathcal{H} \to \mathcal{R}(u,\mathcal{G})$ is a $\mathcal{G}$-**approximation mapping** if it is **separable**, that is, if*

$$[\Phi h]_t(\omega) = [\Phi h']_t(\omega)$$

for all $t \in T, \omega \in \Omega$, and acts $h, h' \in \mathcal{H}$ such that the restrictions of h_t, h'_t to the event $\mathcal{G}_t(\omega) \subset \Omega$ coincide. In addition, $\Phi h = u \circ h$ for every $\mathcal{G}$-adapted act $h \in \mathcal{H}$.

Finally, let μ be a finitely additive probability measure on $\cup_t \mathcal{G}_t$ such that $\mu(A) > 0$ for every nonempty set $A \in \cup_t \mathcal{G}_t$. A list $(u, \beta, \mathcal{G}, \Phi, \mu)$ is a **small-world representation** for a preference relation $\succeq$ on $\mathcal{H}$ if $\succeq$ is represented by the function

$$U(h_0, h_1, ...) = \mathbb{E}_\mu\Big[\sum_t \beta^t [\Phi h]_t\Big].$$

Given a preference relation $\succeq$ on $\mathcal{H}$, let $\{\mathcal{F}_t\}_t$ be the subjective event tree as per Definition C.1. A small-world representation $(u, \beta, \mathcal{G}, \Phi, \mu)$ of $\succeq$ is **proper** if $\mathcal{G}_t = \mathcal{F}_t$ for every t. One can show that a preference relation $\succeq$ has a proper small-world representation if and only if it satisfies axioms analogous to those used in Theorem 2. Theorem 1 and Lemmas 3 and 4 have obvious analogues as well. All formal details can be found in Kochov [6].

E Repeated choice

The remainder of this appendix extends the analysis to the study of repeated choice. As in Section 10, repeated choice is modeled by an $\mathcal{A}$-adapted process $\{\succeq^{t,\omega}\}$ where $\succeq^{t,\omega}$ is a preference relation on $\mathcal{H}$ representing the behavior of the individual in period t if the true state of the world is $\omega \in \Omega$. For the rest of the appendix, fix the process $\{\succeq^{t,\omega}\}$ and assume that it satisfies Conditional Preference and Conditional State Independence.[5]

[4] As is well understood, the infinite horizon setting necessitates a positive rate of time preference, that is, $\beta < 1$.

[5] Given a set C, I define an $\mathcal{A}$-adapted process $\{c^{t,\omega}\}$ to be a collection $\{c^{t,\omega} \in C : t \in T, \omega \in \Omega\}$ such that for every t, the function $\omega \mapsto c^{t,\omega}$ is $\mathcal{A}_t$-measurable. Given such a process, I write c^0 for the function $\omega \mapsto c^{0,\omega}$. Since the function c^0 is constant, one can think of c^0 as an element of the set C.

E.1 Some preliminary notation

Given an event $B \subset \Omega$ and a collection $\mathcal{C} \subset 2^{\Omega}$, let $B \cap \mathcal{C}$ denote the collection $\{B \cap A : A \in \mathcal{C}\}$. Observe that if $\mathcal{C}$ is an algebra on Ω, then $B \cap \mathcal{C}$ is an algebra on B. The filtration $\mathcal{A} = \{\mathcal{A}_t\}_t$ on Ω can be viewed as an event tree and each pair (t, ω) as a node of the tree. Then, each node (t, ω) gives rise to a **continuation tree** $\mathcal{A}^{t,\omega}$ **emanating from that node**. Letting $B := \mathcal{A}_t(\omega)$, the tree $\mathcal{A}^{t,\omega}$ can be formally defined as the sequence

$$\{B \cap \mathcal{A}_t, B \cap \mathcal{A}_{t+1}, ...\}$$

which consists of successively finer algebras on the event $B = \mathcal{A}_t(\omega)$. This sequence is also enumerated as

$$\{\mathcal{A}_t^{t,\omega}, \mathcal{A}_{t+1}^{t,\omega}, \mathcal{A}_{t+2}^{t,\omega}, ...\}.$$

Observe that the first element of $\mathcal{A}^{t,\omega}$ is the trivial algebra on $\mathcal{A}_t(\omega)$:

$$\mathcal{A}_t^{t,\omega} = \mathcal{A}_t(\omega) \cap \mathcal{A}_t = \{\mathcal{A}_t(\omega), \varnothing\}.$$

In particular, $\mathcal{A}^{t,\omega}$ is a filtration on the set $\mathcal{A}_t(\omega)$. In similar fashion, every act $h \in \mathcal{H}$ gives rise to a **continuation act** $h^{t,\omega}$ **emanating from the node** (t, ω). Formally,

$$h^{t,\omega} := (h_t|_{\mathcal{A}_t(\omega)}, h_{t+1}|_{\mathcal{A}_t(\omega)}, ...).$$

In its own right, the continuation act $h^{t,\omega}$ is an X-valued, $\mathcal{A}^{t,\omega}$-adapted process. The space $\{h^{t,\omega} : h \in \mathcal{H}\}$ of all continuation acts emanating from the node (t, ω) is denoted by $\mathcal{H}^{t,\omega}$.

Finally, a **subfiltration** $\mathcal{G}^{t,\omega}$ **of** $\mathcal{A}^{t,\omega}$ is a sequence

$$\{\mathcal{G}_t^{t,\omega}, \mathcal{G}_{t+1}^{t,\omega}, \mathcal{G}_{t+2}^{t,\omega}, ...\}$$

of successively finer algebras on the set $\mathcal{A}_t(\omega)$ such that $\mathcal{G}_k^{t,\omega} \subset \mathcal{A}_k^{t,\omega}$ for every $k \geq t$. Given a subfiltration $\mathcal{G}^{t,\omega}$, I abuse notation slightly and use $\mathcal{G}^{t,\omega}$ to also denote the algebra $\cup_{k \geq t} \mathcal{G}_k^{t,\omega}$ consisting of all the events that comprise the subfiltration $\mathcal{G}^{t,\omega}$.

E.2 What is foreseen at each node?

This section defines what is foreseen at each node (t, ω). To do so, think of each preference relation $\succeq^{t,\omega}$ as a preference relation on the space $\mathcal{H}^{t,\omega}$ of continuation acts. This is possible because I assume Conditional Preference. Then, the problem of defining what is foreseen at (t, ω) is exactly the same as the problem of defining what is foreseen at the initial node. Thus, an analogue of Definition C.1 delivers a sequence

$$\mathcal{F}^{t,\omega} = \{\mathcal{F}_t^{t,\omega}, \mathcal{F}_{t+1}^{t,\omega}, \ldots\}$$

where each collection $\mathcal{F}_k^{t,\omega} \subset \mathcal{A}_k^{t,\omega}, k \in \{t, t+1, \ldots\}$, represents the events that the individual foresees at the node (t, ω) and that he believes to resolve before or in period k.

E.3 Small-world representations again

This section defines a small-word representation

$$(u^{t,\omega}, \beta^{t,\omega}, \mathcal{G}^{t,\omega}, \Phi^{t,\omega}, \mu^{t,\omega}).$$

for each conditional preference $\succeq^{t,\omega}$ in the process $\{\succeq^{t,\omega}\}$. To do so, it is once again convenient to think of each $\succeq^{t,\omega}$ as a preference relation on the space $\mathcal{H}^{t,\omega}$. Then, a small-world representation for $\succeq^{t,\omega}$ can be defined as in Section D. In particular, $\mathcal{G}^{t,\omega}$ becomes a subfiltration of $\mathcal{A}^{t,\omega}$, $\Phi^{t,\omega}$ a function from $\mathcal{H}^{t,\omega}$ into the space of util-denominated, $\mathcal{G}^{t,\omega}$-adapted continuation acts, and $\mu^{t,\omega}$ a probability measure on $\mathcal{G}^{t,\omega}$. Once again, a small-word representation $(u^{t,\omega}, \beta^{t,\omega}, \mathcal{G}^{t,\omega}, \Phi^{t,\omega}, \mu^{t,\omega})$ for $\succeq^{t,\omega}$ is **proper** if $\mathcal{G}^{t,\omega} = \mathcal{F}^{t,\omega}$.

E.4 Relaxing Dynamic Consistency

Theorem 5 shows that unforeseen events are incompatible with Dynamic Consistency. Intuitively, when an individual becomes aware of an event he did not previously foresee, he recognizes that some of his earlier plans are inadequate and should be revised. The next axiom, a relaxation of Dynamic Consistency, requires that earlier plans be revised only when they are actually affected by some previously unforeseen event.

Weak Dynamic Consistency: For all $t \in T, \omega \in \Omega$, and $h, g \in \mathcal{H}$ such that $h_k = g_k$ for all $k \leq t$ and such that the continuation acts $h^{t,\omega}, g^{t,\omega} \in \mathcal{H}^{t,\omega}$ are $\mathcal{F}^{t,\omega}$-adapted,

$$\text{if} \quad h \succeq^{t+1,\omega'} g \quad \text{for all} \quad \omega' \in \mathcal{A}_t(\omega), \quad \text{then} \quad h \succeq^{t,\omega} g.$$

E.5 Updating

Suppose each $\succeq^{t,\omega}$ has a small-world representation $(u^{t,\omega}, \beta^{t,\omega}, \mathcal{G}^{t,\omega}, \Phi^{t,\omega}, \mu^{t,\omega})$. How should the filtrations $\mathcal{G}^{t,\omega}$ and the beliefs $\mu^{t,\omega}$ evolve over time? Regarding the filtrations $\mathcal{G}^{t,\omega}$, it is natural that more events should become foreseen as time unfolds. After all, the individual should not forget events of which he was previously aware. The next definition formalizes what it means for foresight to increase (expand) over time.

Definition E.1 *The process $\{\mathcal{G}^{t,\omega}\}$ is **expanding** if $\mathcal{G}_k^{t+1,\omega} \supset \mathcal{A}_{t+1}(\omega) \cap \mathcal{G}_k^{t,\omega}$ for all $t \in T, \omega \in \Omega$ and $k \geq t+1$.*

Turning to beliefs, I now describe an updating rule proposed by Diaconis and Zabell [1]. Suppose that $\omega \in \Omega$ is the true but unknown state of the world. The interesting case arises when in some period $t \in T$ the individual becomes aware of new events. In particular, it is possible that the event $\mathcal{A}_t(\omega)$ was itself unforeseen so that its realization comes as a surprise. In such circumstances, the first job of the individual is to assign probabilities to the new events he is aware of. Formally, this means that the probability measure $\mu^{t-1,\omega}$ is extended from the algebra $\mathcal{G}^{t-1,\omega}$ on the event $\mathcal{A}_{t-1}(\omega)$ to the coarsest algebra on $\mathcal{A}_{t-1}(\omega)$ that includes both the events in $\mathcal{G}^{t-1,\omega}$ and the newly foreseen events in $\mathcal{G}^{t,\omega}$. This extension is **consistent** in that the individual does not revise the prior likelihoods of any events in $\mathcal{G}^{t-1,\omega}$. Diaconis and Zabell [1] refer to this step as the **retrospective construction of a prior**. Once beliefs are extended in this manner, the individual updates the extended belief by Bayes' rule.

The above description shows how updating plays out in real time. It turns out that there is an alternative and more succinct way to describe the same updating rule. Focus on the fact that foresight is continually expanding over time. Since, in addition, beliefs are consistently extended at each stage, it is reasonable to conjecture that in the limit this process will result in a probability measure $\hat{\mu}$ defined on the entire algebra $\cup_t \mathcal{A}_t$. This is indeed the case. Mathematically, one can think of $\hat{\mu}$ as the projective limit of the process $\{\mathcal{G}^{t,\omega}, \mu^{t,\omega}\}$. A more revealing but somewhat less formal interpretation is to think of $\hat{\mu}$ as the prior belief the individual would have had if he could foresee all events at the beginning of time. Regardless of the interpretation, the consistent extension $\hat{\mu}$ provides a convenient way to compute the beliefs $\mu^{t,\omega}$ of the individual at any given node: First, find the Bayes' posterior of $\hat{\mu}$ and, then, restrict this posterior to the collection of events foreseen at the given node.

Definition E.2 *The process* $\{\mathcal{G}^{t,\omega}, \mu^{t,\omega}\}$ ***admits a consistent extension*** $\hat{\mu}$, *where* $\hat{\mu}$ *is a probability measure on* $(\Omega, \cup_t \mathcal{A}_t)$ *such that* $\hat{\mu}(A) > 0$ *for every nonempty set* $A \in \cup_t \mathcal{A}_t$, *if*

$$\mu^{t,\omega}(A) = \hat{\mu}(A|\mathcal{A}_t(\omega)) \quad \textit{for every} \quad t \in T, \omega \in \Omega, A \in \mathcal{G}^{t,\omega}.$$

The next theorem shows that, under a mild auxiliary assumption, Weak Dynamic Consistency holds if and only if foresight is expanding and beliefs are updated in the manner of Definition E.2.

Theorem E.1 *Suppose each preference relation* $\succeq^{t,\omega}$ *has a small-world representation* $(u, \beta, \mathcal{F}^{t,\omega}, \Phi^{t,\omega}, \mu^{t,\omega})$. *Suppose at each point in time the individual foresees all one-step-ahead contingencies, that is,* $\mathcal{A}_{t+1}(\omega) \in \mathcal{F}^{t,\omega}_{t+1}$ *for all* $t \in T, \omega \in \Omega$. *Then, the process* $\{\succeq^{t,\omega}\}$ *satisfies Weak Dynamic Consistency if and only if the process* $\{\mathcal{F}^{t,\omega}\}$ *is expanding and* $\{\mu^{t,\omega}, \mathcal{F}^{t,\omega}\}$ *admits a consistent extension* $\hat{\mu}$. *Moreover, the extension* $\hat{\mu}$ *is unique.*[6]

The proof is long but contains few surprising elements. The assumption that the individual foresees all one-step-ahead contingencies can be replaced by the assumption that each filtration $\mathcal{F}^{t,\omega}$ is a stopping time. Once again, all details can be found in Kochov [6]. In terms of related literature, a similar rule for updating beliefs has been independently axiomatized by Hayashi [3] and Karni and Vierø [5]. One important difference is that these papers take what is foreseen as exogenously given. Then, they ask the question: what happens to beliefs if foresight were to expand? In this paper, what is foreseen is endogenous. As in Theorem E.1, one must therefore identify the appropriate restrictions on behavior under which foresight does indeed expand.

Outside of the decision-theoretic literature, the idea that foresight should be expanding over time has been utilized in Rêgo and Halpern [8] and Halpern and Rêgo [2] in order to study extensive form games with unawareness. The idea that beliefs should be consistently extended as the individual becomes aware of more events has been utilized in the work of Heifetz, Meier, and Schipper [4] and Modica [7]. Theorem E.1 complements these papers by providing explicit choice-theoretic foundations for such ideas.

[6] Though not explicitly stated, the axioms Conditional Preference and Conditional State Independence are implicit in the assumption that each $\succeq^{t,\omega}$ has a small-world representation. Thus, note that for $t > 0$ such representations were defined only when Conditional Preference holds. As for Conditional State Independence, note that $u : X \to \mathbb{R}$ and the discount factor $\beta \in (0, 1)$ are not indexed by t and ω.

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